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Kwiatkowski, Andrzej

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Education investment effects of affirmative action policy. Contest game argument.

Andrzej Kwiatkowski

Department of
Economic Studies
University of Dundee
Dundee
DD1 4HN

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Education investment effects of affirmative action policy. Contest game argument.

Andrzej Kwiatkowski
Economic Studies, University of Dundee*

Abstract

In this paper we investigate the problem of effort effects of the affirmative action policy. We develop a version of a rent-seeking game in the style of Tullock (1980) with two heterogeneous players and two stages, considering that ahead of the formal competition players can invest to lower their effort cost. Using this model we show that there are instances in which the normative objective of affirmative action policy to make a level-playing field may be missed. Namely, we demonstrate that if in relative terms the cost of acquiring skills for the *ex ante* weaker player (a member of a discriminated group) is low enough as compared to the *ex ante* stronger player (non-discriminated), then in the actual competition the *ex ante* weaker player may become stronger than the *ex ante* stronger player. This result shows that AA programmes cannot be effective if they are designed in isolation based on the minority-group membership only and without taking into account the actual costs of acquiring skills (that is education or learning) by individuals.

Keywords: Asymmetric contest; afirmative action; discrimination; education;
JEL classification: C72; D63; I38; J78

1 Introduction

Affirmative action (called also positive discrimination) is a public policy instrument whose objective is to ameliorate the adverse effects of discrimination on affected groups of individuals. One of the potential consequences of affirmative action programs is with respect to effort incentives, affecting the effort levels of both – the discriminated and non-discriminated individual. The problem of effort provision under equal treatment and affirmative action policy in competitive settings, such as in sports, workplaces, university/school admissions, etc., is addressed by many authors (see for instance Fu (2006); Schotter and Weigelt (1992), Fain (2009), Franke (2012), Nti (2004) and Runkel

*Correspondence: Economic Studies, University of Dundee, 3 Perth Road, Dundee DD1 4HN, UK; Tel: (+44) (0) 1382 384372; Fax: (+44) (0) 1382 384691; Email: a.kwiatkowski@dundee.ac.uk.

(2006)) and their research provides certain guidance on the effort effects of those policy instruments. A common feature of those studies is that all the effort investment is made in the competition stage only. In reality however, such contests are often preceded by investments of the players into their ability to fight in the competition. In this paper we study the problem of effects of the affirmative action policy, considering that ahead of the formal competition players can invest to lower their effort cost.

There is a handful of theoretical economic studies on contest-like situations that incorporate the idea of costly investment made by players to improve competency prior to the formal competition (see for instance Münster (2007) or Fu and Lu (2009)). They consider games with two stages where players expend two separate types of effort: the effort in the first stage to improve their skills to fight in the competition and the effort in the second stage in the actual competition. Formally, players' skills to fight in the competition are reflected by their marginal cost of effort, and any investment in the effort in the first stage reduces its level. In this paper we build on this idea. We assume that prior to the competition players may make an effort investment, costly in terms of their utility, to reduce their marginal cost of effort in the fight in that competition. We will call this type of effort "learning effort", to distinguish it from the from the type of effort individuals exert exclusively in the competition stage - "competitive effort"[1]. This distinction between the two types of effort may be very important if we consider the effects of affirmative action programmes, as learning effort may respond to various incentives in a way different than competitive effort[2].

We develop a simple model which is a version of a rent-seeking game in the style of Tullock (1980) with two heterogeneous players and two stages. *Ex ante,* players differ in their levels of marginal cost of (competitive) effort. The player with a higher level of that marginal cost *ex ante* is considered to be discriminated and the other one - non-discriminated. Prior to the actual competition there is a learning stage in which players can invest some (learning) effort to lower their own levels of marginal cost of (competitive) effort. Exerting learning effort, however, is costly in terms of utility, and

[1] The distinction between different types of effort investmenst is motivated not only by the observation that in real life settings learning process is separated in time from actual competition. What also makes learning distinct from actual competing is the fact that learning investment may require some other skills and have other nature than actual competing and is also usually more spread in time. Learning is often a repetitive task or process, whereas actual competing can be viewed as a kind of one-off task. Moreover, "learning effort" can be interpreted in much broader sense as any costly investements improving the skills level (or human capital) of individuals. In this way, purchases of any essential equipment required to learn or study or any other capital investemnt in human capital can be viewed as a form of "learning effort", as defined in our model.

[2] As a result of that potentially it may happen that an *ex ante* weaker (less skilled, discriminated) agent becomes the stronger one (more skilled) in the actual competition. This means that the objective of the affirmative action policy to ameliorate the adverse effects of discrimination and creating a level playing field for all agents is totally missed. That is in the actual competition the discriminated agent is more skilled than the non-discriminated one and in addition benefits from the affirmative action policy support.

the players differ in their marginal cost of learning effort[3]. In the model we consider two policy options: equal treatment policy and affirmative action policy. The affirmative action policy can vary in its intensity and the equal treatment option is defined as a special case of the affirmative action policy option when its intensity is zero. The policy(ies) are defined formally as restrictions on the contest rule which, depending on the implemented policy, imply different effort incentives for the individuals. With this formulation, the key question that we study is how individuals react to the changes in incentives that are induced by the policies as regards their learning and competitive effort.

Using our model we demonstrate that allowing the players to invest in their learning effort has a strong effect on their levels of the marginal cost of the competitive effort in the actual competition, and that in equilibrium the exact levels of that marginal cost are influenced by the intensity level of the implemented AA policy. Over the wealth of the parameter space, for each agent the relation between the learning effort level and the intensity of the AA policy follows an inverted-U-shaped curve, with one distinct maximum in the intensity level. Interestingly, we observe that for some combinations of the model parameters the objective of the AA policy to create a level playing field, is totally missed: in equilibrium in the final stage of the game the *ex ante* weaker (discriminated) player actually becomes stronger than the *ex ante* stronger (non-discriminated) one. This happens when in the relative terms the cost of learning for the discriminated player is low enough as compared to the cost of learning of the non-discriminated player. This result shows that AA programmes implemented in the formal competition cannot be effective if they are designed in isolation based on the minority-group membership only and without taking into account the actual costs of acquiring skills (that is education or learning) by individuals. Using our model we also investigate the problem of the optimality of the contest from the point of view of the contest holder utilizing as the standard of comparison of various policies the total level of the learning effort and the total level of the competitive effort. We show that for both standard of comparison there exist an optimal level of the intensity level of the implemented AA policy, including the ET policy, that induces that highest level of effort In general this maximum requires some degree of the intensity of the AA policy. However, there are also cases when the maximum occurs for when the ET policy is implemented.

Our work is related to some other models in the economic theory literature. In particular it is related to numerous papers that study the effects of affirmative action policy in various competitive settings. In this strand of research, Franke (2012) investigates the problem of effort provision under equal treatment and affirmative action policy. Fryer and Loury (2005) consider a simple model of pair-wise tournament competition to investigate group-sighted and group-blind forms of affirmative action

[3]In this dimension, our model is a form of generalization of Münster (2007) and Fu and Lu (2009) mentioned earlier, who assumed that the marginal cost of learnig effort of individuals is the same. By assuming heterogeneity in this respect we are able to investigate the role played by differences in the cost of aquiring skills as regards the levels of both types of effort and how they interplay with the effort incentives of affirmative action policy.

in winner-take-all-markets. Fain (2009) and Schotterand and Weigelt (1992) using the tournament-game framework study whether affirmative action programs and equal opportunity laws affect the output of economic agents. Fu (2006) addresses the problem of affirmative action policy in admissions to a college using a two-player all-pay auction model. Our work is related to numerous works on contest games in various settings[4]. In this strand of the literature the closest to our study are Münster (2007) and Fu and Lu (2009) who consider the contests in which players prior to the formal competition make costly investment to improve their skills, and Nti (2004) and Runkel (2006), who investigate the problem of maximum level of total effort in contests.

This paper is organized as follows: In Section 2 we introduce our model. In Section 3 we compute the model equilibrium and perform the analysis of learning effort and the parameters of marginal cost of competitive effort. Section 4 concludes.

2 The model

Typical settings where affirmative action instruments are employed cover various situations of competitive social interaction. The competitive structure of these situations can be conveniently captured using a contest game model. In this game contestants compete for an indivisible prize by exerting (competitive) effort, and by exerting more effort the contestants can increase their respective probability of winning the prize. This feature of the game reflects the basic structure of many situations of competitive social interaction. Another property reflected by a contest game model is that contestants face a probabilistic outcome, being the consequence of a relatively high grade of discretion on the side of the competition organizer. A distinguishing feature of our model relative to those already existing in the literature lies in the assumption on the existence of two types of effort: competitive and learning, exerted exclusively in two different stages of the game. Competitive effort is exerted in the competitive stage when the actual fight for the prize takes place, whereas learning effort is exerted in the learning stage before the fight. The purpose of exerting learning effort is to lower the marginal cost of competitive effort.

To guarantee analytical tractability and closed form solutions, our model is formulated under complete information, i.e. the only element of uncertainty is the final winner of the contest. In the paper we will use the standard notion of Subgame Perfect Equilibrium (SPE).

2.1 Primitives

Let $N = \{1, 2\}$ denote the set of risk-neutral individuals who compete against each other in a contest game. Agent 1 is assumed to be a member of a non-discriminated group (we will call them a non-discriminated player), and agent 2 – a member of a discriminated group (a discriminated player). The contest is organized in the following way: It consists of two stages $S = \{0, 1\}$. In stage 1 the agents compete against each

[4] For a more detailed review of the literature in this area of research see Nitzan (1994) or Dechenaux, Kovenock and Sheremeta (2014).

other and the winner gets the prize. To win the contest, each contestant $i \in N$ exerts a competitive effort level $e_i \in R^+$, while their opponent – a contestant $j \in N, i \neq j$ – exerts a competitive effort level $e_j \in R^+$. It is assumed that both contestants have the same positive valuation V for the contested prize. The contestants differ in the respective "cost function" that captures the disutility of exerting competitive effort e_i. It is assumed that for all $i \in N$ this cost function is linear in e_i and multiplicative in β_{1i}, such that:

$$c_i(e_i) = \beta_{1i} e_i, \tag{1}$$

where $\beta_{1i} > 0$ is the parameter of marginal cost of competitive effort of an agent i in stage 1. By assumption, marginal cost of competitive effort is finite and constant. In stage 0 the agents may decide to invest some effort into learning. At the beginning of the game an agent i has initial marginal cost of competitive effort *ex ante*, $\beta_{0i} > 0$. By investment in learning in stage 0 an agent i may reduce the level of that marginal cost. Formally, given a learning effort level $\epsilon_i \in R^+$ and the marginal cost of competitive effort *ex ante*, β_{0i}, the marginal cost of competitive effort of an agent i becomes

$$\beta_{1i} \equiv \beta_{1i}(\epsilon_i) = \frac{\beta_{0i}}{\sqrt{1+\epsilon_i}}^{5}. \tag{2}$$

Exerting learning effort is costly in terms of utility. The contestants differ in the cost function that captures the disutility of exerting learning effort ϵ_i. It is assumed that for all $i \in N$ this cost function is linear in ϵ_i and multiplicative in γ_i, such that:

$$c_i(\epsilon_i) = \gamma_i \epsilon_i, \tag{3}$$

where $\gamma_i > 0$ is the parameter of marginal cost of [6] learning effort of an agent i in stage 0. By assumption, marginal cost of the learning effort is finite and constant.

We assume also that the contestants are heterogenous in terms of their parameter of marginal cost of competitive effort *ex ante* β_{0i} and are ordered, such that $\beta_{01} < \beta_{02}$, with normalization $\beta_{01} = 1$. We denote $\beta_{02} = \beta$.

The contestants perceive the outcome of the contest stage of the game as probabilistic. However, they can influence the probability of winning by exerting competitive effort, which means that the outcome depends on the vector of the effort levels exerted by both individuals. In our model we will employ the following Contest Success Function (CSF) $p_i : R^2_+ \rightarrow [0, 1]$:

$$p_i(e_i, e_j) = \frac{\alpha_i^P e_i}{\alpha_i^P e_i + \alpha_j^P e_j}, \text{ for all } i \in N, \tag{4}$$

with $\alpha_i^P > 0$ for all $i \in N$. This function maps the vector of effort levels (e_i, e_j) into win probabilities for each contestant. This is a restricted version of a CSF axiomatized in

[6] As noted ealier, in this dimension, our model is a form of generalization of Münster (2007) and Fu and Lu (2009), who assumed that the marginal cost of learnig effort of individuals is the same.

Clark and Riis (1998)[7]. This function possesses a very convenient feature that allows an asymmetric treatment of the contestants that can be interpreted as affirmative action policy. This is done by appropriate setting values of positive weights α_i^P, that depend on the policy P. If no contestant exerts positive effort, it is assumed that none of the individuals receives the prize, i.e. $p_i(0,0) = 0$ for all $i \in N$[8].

A contestant $i \in N$ aims to maximize his expected utility, which, given the cost functions (1) and (3) and the contest mechanism (4), takes the following (additive separable) form:

$$u_i\left(e_i, e_j, \epsilon_i\right) = p_i(e_i, e_j)V - \beta_{i1}\left(\epsilon_i\right) e_i - \gamma_i \epsilon_i. \tag{5}$$

We assume that in both stages while making their effort decision the agents behave in a non-cooperative way. The implemented policy option P is announced to both contestants before the whole game starts in stage 0.

2.2 The Policy Options

In this section we describe the set of policy options P which later will be compared in terms of learning effort levels that they generate. Agent 1 is a member of a non-discriminated group and agent 2 – of a discriminated group. With respect to this discrimination, we consider two policy options: equal treatment policy (ET) and affirmative action policy (AA). Equal treatment policy requires that both players are treated in the same way, independently of whether they are members of a discriminated or non-discriminated group. In turn, under affirmative action policy the discriminated group member obtains some advantage with respect to the non-discriminated group member.

Formally, in our model the two policy options are reflected by setting appropriately the individual competitive effort weights (α_i^P, α_j^P) in the CSF defined in (4). Under equal treatment policy (ET) both players are treated by the contest rule (4) in the same symmetric way. That is, if both players exert their competitive effort at the same level, then each of them has winning probability of one half. This implies that policy weights must be equal for all players, that is

$$\alpha_i^{ET} = \alpha^{ET} \text{ for all } i \in N.$$

Given that the CSF is homogenous of degree zero, without loss of generality these

[7] In Clark and Riis (1998) the CSF has the form $p_i(e_i, e_j) = \frac{\alpha_i^P e_i^r}{\alpha_i^P e_i^r + \alpha_j^P e_j^r}$, for all $i \in N$, with $r > 0$. The parameter r measures the sensitivity of the outcome of the contest game with respect to differences in effort. The assumption about r is needed because for a non-linear CSF with a general parameter $r > 0$ it is not possible to derive closed form solutions. As the existence of closed form solutions is crucial for the comparative analysis of the policy alternatives, it is assumed that the CSF is linear with $r = 1$. Also with a general parameter $r > 0$ the existence of pure strategy equilibria cannot be guaranteed (see Baye, Kovenock and de Vries (1994) for details). With the restriction $r = 1$ all our equilibria are in pure strategies.

[8] Another convention in the contest-game literature is that $p_i(0,0) = \frac{1}{2}$ for all $i \in N$. The choice of either definition is not important in terms of our results.

weights can be normalized such that

$$\alpha_i^{ET} = 1 \text{ for all } i \in N.$$

In turn, under affirmative action policy the discriminated group member is favoured by the contest rule (4), which increases their winning probability compared to the non-discriminated group member. That is, if both players exert the same level of the competitive effort, then the discriminated group player wins with probability higher than one half, and the non-discriminated group member with probability less than one half. Therefore the AA policy generates a bias of the CSF in favor of the discriminated contestant. This implies that policy weights must satisfy

$$\alpha_1^{AA} < \alpha_2^{AA}.$$

Given that the CSF is homogenous of degree zero, without loss of generality these weights can be normalized such that

$$\alpha_1^{AA} = 1 < \alpha_2^{AA}.$$

The equilibrium effort levels of each contestant will depend on the *ex ante* announced policy parameter P. The main focus of our analysis will be the effects of the policy options on the levels of learning effort and the resulting marginal cost of competitive effort β_{1i}.

Using these results and setting $\alpha = \alpha_2^P$ we can rewrite the CSF defined in (4) as

$$\begin{cases} p_1(e_1, e_2) = \frac{e_1}{e_1 + \alpha e_2}, \\ p_2(e_1, e_2) = \frac{\alpha e_2}{e_1 + \alpha e_2}. \end{cases} \tag{6}$$

where

$$\begin{cases} \alpha = 1, \text{if } P = ET, \\ \alpha > 1, \text{if } P = AA. \end{cases}$$

Note that the definition of the policy parameter α allows us to study the effects of implementation of various instances of affirmative action policy, differing in their intensity. It is also worth noting that with this formulation of the CSF and the policy options equal treatment policy (ET) can be interpreted as a particular case of affirmative action policy (AA) for when its intensity is zero.

3 Analysis

3.1 Stage 1

We start by solving our model by backward induction for competitive effort levels of players in stage 1. In this stage agents 1 and 2 compete against each other by exerting competitive effort. Plugging the CSF and the cost function as specified in eq. (6)

and (1) into the expected utility function of an agent i in eq. (5) and differentiating, produces the following first order conditions:

$$\begin{cases} \frac{\alpha e_2}{(e_1+\alpha e_2)^2} V - \beta_{11} = 0, \text{ for agent 1,} \\ \frac{\alpha e_1}{(e_1+\alpha e_2)^2} V - \beta_{12} = 0, \text{ for agent 2,} \end{cases}$$

which after some algebra yields the (sub)equilibrium effort level candidates:

$$\begin{cases} e_1 = \frac{\alpha\beta_{12}}{(\alpha\beta_{11}+\beta_{12})^2} V, \\ e_2 = \frac{\alpha\beta_{11}}{(\alpha\beta_{11}+\beta_{12})^2} V. \end{cases}$$

Those competitive effort level candidates are strictly positive, given our assumptions on the parameters. The second order conditions can be expressed as

$$\begin{cases} \frac{\partial^2 u_1(e_1,e_2,\epsilon_1)}{\partial e_1^2} = -\frac{2\alpha e_2}{(e_1+\alpha e_2)^3} V < 0, \\ \frac{\partial^2 u_2(e_1,e_2,\epsilon_2)}{\partial e_2^2} = -\frac{2\alpha^2 e_1}{(e_1+\alpha e_2)^3} V < 0, \end{cases}$$

which proves concavity. Thus the maxima exists and are interior and unique.

So it follows from our analysis, that there exists a unique interior (sub)equilibrium, in which players exert competitive effort at positive levels. Those equilibrium effort levels are

$$\begin{cases} e_1^* = \frac{\alpha\beta_{12}}{(\alpha\beta_{11}+\beta_{12})^2} V, \\ e_2^* = \frac{\alpha\beta_{11}}{(\alpha\beta_{11}+\beta_{12})^2} V. \end{cases} \tag{7}$$

Plugging this result into eq. (5) (together with the CSF and the cost function as specified in equations (6) and (1) produces the (sub)equilibrium expected payoffs

$$\begin{cases} u_1\left(e_1^*, e_2^*, \epsilon_1\right) = \frac{\beta_{12}^2}{(\alpha\beta_{11}+\beta_{12})^2} V - \gamma_1\epsilon_1, \\ u_2\left(e_1^*, e_2^*, , \epsilon_2\right) = \frac{\alpha^2\beta_{11}^2}{(\alpha\beta_{11}+\beta_{12})^2} V - \gamma_2\epsilon_2. \end{cases} \tag{8}$$

3.2 Stage 0

In stage 0 each agent chooses their level of learning effort. That effort allows the individuals to modify their respective parameters of marginal cost of competitive effort in stage 2. The problem of agent 1 in this stage is to maximize their expected payoff $u_1\left(e_1^*, e_2^*, \epsilon_1\right)$ given in eq. (8) with respect to their non-negative learning effort level ϵ_1, given a non-negative learning effort level of agent 2, ϵ_2. In a similar way, agent 2 maximizes their expected payoff $u_2\left(e_1^*, e_2^*, , \epsilon_2\right)$ with respect to their non-negative learning effort level ϵ_2, given a non-negative learning effort level of agent 1, ϵ_1.

After plugging the relation between a learning effort level ϵ_i and the marginal cost of competitive effort *ex ante* β_{0i} as specified in eq. (2) into the expected payoff functions in eq. (8), considering that $\beta_{01} = 1$ and $\beta_{02} = \beta$ and differentiating, the first order conditions yield

$$\begin{cases} \frac{\alpha\beta^2\sqrt{1+\epsilon_2}}{\left(\alpha\sqrt{1+\epsilon_2}+\beta\sqrt{1+\epsilon_1}\right)^3} V - \gamma_1 = 0, \text{ for agent 1,} \\ \frac{\alpha^2\beta\sqrt{1+\epsilon_1}}{\left(\alpha\sqrt{1+\epsilon_2}+\beta\sqrt{1+\epsilon_1}\right)^3} V - \gamma_2 = 0, \text{ for agent 2.} \end{cases} \tag{9}$$

Considering the non-negativity constraints those formulas produce the following best reply functions:

$$\begin{cases} B_1(\epsilon_2) = \max\left\{ \frac{(1+\epsilon_2)\alpha^2\gamma_1 + \beta^{4/3}(V\alpha)^{2/3}((1+\epsilon_2)\gamma_1)^{1/3} - 2(1+\epsilon_2)^{2/3}\left(V\alpha^4\beta^2\gamma_1^2\right)^{1/3}}{\beta^2\gamma_1} - 1, 0 \right\}, \\ B_2(\epsilon_1) = \max\left\{ \frac{(1+\epsilon_1)\beta^2\gamma_2 + \alpha^{4/3}(V\beta)^{2/3}((1+\epsilon_1)\gamma_2)^{1/3} - 2(1+\epsilon_1)^{2/3}\left(V\alpha^2\beta^4{\gamma_2}^2\right)^{1/3}}{\alpha^2\gamma_2} - 1, 0 \right\}. \end{cases} \tag{10}$$

To find equilibrium for learning effort we study four cases:

Case 1: $B_1(\epsilon_2) > 0$ and $B_2(\epsilon_1) > 0$,
Case 2: $B_1(\epsilon_2) > 0$ and $B_2(\epsilon_1) = 0$,
Case 3: $B_1(\epsilon_2) = 0$ and $B_2(\epsilon_1) > 0$,
Case 4: $B_1(\epsilon_2) = 0$ and $B_2(\epsilon_1) = 0$.

Case1: $B_1(\epsilon_2) > 0$ **and** $B_2(\epsilon_1) > 0$ Solving the system of equations formed by the corresponding best reply formulas in (10) we obtain the following equilibrium effort level candidates

$$\begin{cases} \epsilon_1 = \frac{\alpha^2\beta^4\gamma_2^2}{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3} V - 1, \\ \epsilon_2 = \frac{\alpha^4\beta^2\gamma_1^2}{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3} V - 1, \end{cases}$$

which are both strictly positive if the following condition holds

$$V > \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^2\beta^4{\gamma_2}^2} \cap V > \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^4\beta^2{\gamma_1}^2}.$$

Case 2: $B_1(\epsilon_2) > 0$ **and** $B_2(\epsilon_1) = 0$ Setting $\epsilon_2 = B_2(\epsilon_1) = 0$ in $B_1(\epsilon_2)$ in the first line of (10) yields the following equilibrium effort level candidate

$$\epsilon_1 = \frac{\alpha^{\frac{2}{3}}\left(V^{\frac{1}{3}}\beta^{\frac{2}{3}} - \alpha^{\frac{2}{3}}\gamma_1^{\frac{1}{3}}\right)^2}{\beta^2\gamma_1^{\frac{2}{3}}} - 1,$$

which is strictly positive whenever the following condition holds

$$\left(\frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^2\beta^4{\gamma_2}^2} < V \le \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^4\beta^2{\gamma_1}^2}\right) \cap \frac{\alpha}{\beta} < \frac{\gamma_2}{\gamma_1} \cap \left(\left(\alpha > \beta \cap V < \frac{(\alpha-\beta)^3\gamma_1}{\alpha\beta^2}\right) \cup V > \frac{(\alpha+\beta)^3\gamma_1}{\alpha\beta^2}\right)$$

Case 3: $B_1(\epsilon_2) = 0$ **and** $B_2(\epsilon_1) > 0$ Setting $\epsilon_1 = B_1(\epsilon_2) = 0$ in $B_2(\epsilon_1)$ in the second line of (10) yields the following equilibrium effort level candidate

$$\epsilon_2 = \frac{\beta^{\frac{2}{3}}\left(V^{\frac{1}{3}}\alpha^{\frac{2}{3}} - \beta^{\frac{2}{3}}\gamma_2^{\frac{1}{3}}\right)^2}{\alpha^2\gamma_2^{\frac{2}{3}}} - 1,$$

which is strictly positive whenever the following condition holds

$$\left(\frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^4\beta^2{\gamma_1}^2} < V \le \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^2\beta^4{\gamma_2}^2}\right) \cap \frac{\gamma_1}{\gamma_2} > \frac{\beta}{\alpha} \cap \left(\left(\alpha < \beta \cap V < \frac{(\beta-\alpha)^3\gamma_2}{\alpha^2\beta}\right) \cup V > \frac{(\alpha+\beta)^3\gamma_2}{\alpha^2\beta}\right)$$

Case 4: $B_1(\epsilon_2) = 0$ **and** $B_2(\epsilon_1) = 0$ In this case both equilibrium effort level candidates are zero. This happens whenever none of the previous conditions for positive effort levels holds.

The second order conditions can be expressed as

$$\begin{cases} \frac{\partial^2 u_1(e_1,e_2,\epsilon_1)}{\partial \epsilon_1^2} = -\frac{3\alpha\beta^3\sqrt{1+\epsilon_2}}{2\sqrt{1+\epsilon_1}\left(\alpha\sqrt{1+\epsilon_2}+\beta\sqrt{1+\epsilon_1}\right)^4}V < 0, \\ \frac{\partial^2 u_2(e_1,e_2,\epsilon_2)}{\partial e_2^2} = -\frac{3\alpha^3\beta\sqrt{1+\epsilon_1}}{2\sqrt{1+\epsilon_2}\left(\alpha\sqrt{1+\epsilon_2}+\beta\sqrt{1+\epsilon_1}\right)^4}V < 0, \end{cases}$$

which proves concavity. Thus the maxima exist and are unique.

So it follows from our analysis, that there exists a unique (sub)equilibrium, in which the equilibrium learning effort levels are defined in the following way:

$$(\epsilon_1^*, \epsilon_2^*) = \begin{cases} \left(\frac{\alpha^2\beta^4\gamma_2^2}{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}V - 1, \frac{\alpha^4\beta^2\gamma_1^2}{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}V - 1\right), \text{if} & C1, \\ \left(\frac{\alpha^{2/3}\left(\left(V\beta^2\right)^{1/3}-\left(\alpha^2\gamma_1\right)^{1/3}\right)^2}{\beta^2\gamma_1^{2/3}} - 1, 0\right), \quad \text{if} & C2, \\ \left(0, \frac{\beta^{2/3}\left(\left(V\alpha^2\right)^{1/3}-\left(\beta^2\gamma_2\right)^{1/3}\right)^2}{\alpha^2\gamma_2^{2/3}} - 1\right), \quad \text{if} & C3, \\ (0,0), & \text{otherwise}, \end{cases} \tag{11}$$

where

$$C1 = \left\{V > \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^2\beta^4{\gamma_2}^2} \cap V > \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^4\beta^2{\gamma_1}^2}\right\},$$

$$C2 = \left\{\left(\frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^2\beta^4{\gamma_2}^2} < V \leq \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^4\beta^2{\gamma_1}^2}\right) \cap \frac{\gamma_1}{\gamma_2} < \frac{\beta}{\alpha} \cap \left(\left(\alpha > \beta \cap V < \frac{(\alpha-\beta)^3\gamma_1}{\alpha\beta^2}\right) \cup V > \frac{(\alpha+\beta)^3\gamma_1}{\alpha\beta^2}\right)\right\},$$

$$C3 = \left\{\left(\frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^4\beta^2{\gamma_1}^2} < V \leq \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^2\beta^4{\gamma_2}^2}\right) \cap \frac{\gamma_1}{\gamma_2} > \frac{\beta}{\alpha} \cap \left(\left(\alpha < \beta \cap V < \frac{(\beta-\alpha)^3\gamma_2}{\alpha^2\beta}\right) \cup V > \frac{(\alpha+\beta)^3\gamma_2}{\alpha^2\beta}\right)\right\}.$$

Plugging this result into eq. (2) and considering that $\beta_{01} = 1$ and $\beta_{02} = \beta$, produces the (sub)equilibrium levels of marginal effort of competitive effort of players

$$(\beta_{11}^*, \beta_{12}^*) = \begin{cases} \left(\frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^{3/2}}{\sqrt{V}\alpha\beta^2\gamma_2}, \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^{3/2}}{\sqrt{V}\alpha^2\gamma_1}\right), \text{if} & C1, \\ \left(\frac{\beta\gamma_1^{1/3}}{\alpha^{1/3}\sqrt{\left(\left(V\beta^2\right)^{1/3}-\left(\alpha^2\gamma_1\right)^{1/3}\right)^2}}, \beta\right), \text{if} & C2, \\ \left(1, \frac{\alpha\left(\beta^2\gamma_2\right)^{1/3}}{\sqrt{\left(\left(V\alpha^2\right)^{1/3}-\left(\beta^2\gamma_2\right)^{1/3}\right)^2}}\right), \quad \text{if} & C3, \\ (1, \beta), & \text{otherwise.} \end{cases} \quad (12)$$

Conditions C1 - C3 define the sets of parameters' values for when the learning effort levels of players are positive. It is worth discussing here the role played by the value of the contest prize V in determining whether those are both, just one, or none of the players who exert their learning effort at a positive level. When the prize value is very high and the first part of condition C1 holds for agent 1 ($V > \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^2\beta^4{\gamma_2}^2}$) and the second for agent 2 ($V > \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^4\beta^2{\gamma_1}^2}$), then both players improve their marginal cost of competitive effort by exerting learning effort at a positive level. However, when V is at a moderate level, then either agent 1 or agent 2 stops investing in their learning. Whether this is specifically agent 1 or agent 2, it depends first of all on whether the first part or the second part of condition C1 still holds. If the first part of C1 holds, but not the second one (that is $\frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^2\beta^4{\gamma_2}^2} < V \leq \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^4\beta^2{\gamma_1}^2}$), then agent 1 may be active in exerting their learning effort (subject to some extra conditions) and agent 2 is not active. In turn if the first part of C1 doesn't hold, but the second one does (that is $\frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^4\beta^2{\gamma_1}^2} < V \leq \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3}{\alpha^2\beta^4{\gamma_2}^2}$), then agent 1 is not active in exerting their learning effort and agent 2 may be active (again subject to some extra conditions). It's worth noting here that when the prize value V is very low, then none of conditions C1 - C3 is satisfied and none of the players invests in their learning effort.

Careful investigation of conditions C1 - C3 reveals that whether this is the first part or the second part of condition C1 that holds depends on the relation between $\frac{\gamma_1}{\gamma_2}$ and $\frac{\beta}{\alpha}$. The first ratio is the relative marginal cost of learning effort of agent 1 with respect to agent 2, the second one measures in relative terms the size of the discrimination level of agent 2 as reflected by β, with respect to the extent of the affirmative action policy, α. For the first part of C1 to hold and not the second one, that is for agent 1 being active in exerting their learning effort and agent 2 inactive it must be the case that

$$\frac{\gamma_1}{\gamma_2} < \frac{\beta}{\alpha}.$$

This condition will hold whenever the intensity of the affirmative action policy is relatively low as compared to the level of discrimination of agent 2 *ex ante* β and/or when the marginal cost of learning effort of agent 2 γ_2 is relatively high as compared to the marginal cost of learning effort of agent 1 γ_1, that when is learning is costly in relative terms for the discriminated player. In turn, for the first part of C1 not to hold and the second one to hold, that is for agent 1 being inactive and agent 2 active it must be the case that

$$\frac{\gamma_1}{\gamma_2} > \frac{\beta}{\alpha},$$

which will hold when the intensity of the affirmative action policy is relatively high as compared to the level of discrimination of agent 2 *ex ante* β and/or when the marginal cost of learning effort of agent 2 γ_2 is relatively low as compared to the marginal cost of learning effort of agent 1, γ_1, that is when learning is cheap in relative terms for the discriminated player.

So putting aside all the additional conditions in C2 and C3 required for one agent only to be active in exerting learning effort, it can be noted that a crucial role in determining whether this is either agent 1 or 2 who is active is played by the intensity of the affirmative action policy and/or how costly it is for the discriminated player to invest in learning effort. Cheap learning for agent 2 and/or a high intensity of the AA policy makes it more likely for the discriminated player to invest. These incentives disappear if learning is costly and/or if there is low intensity of the AA policy.

The additional conditions in C2 for active agent 1 when agent 2 is inactive require that either the intensity of affirmative action policy is high enough (higher in value than the marginal cost of competitive effort *ex ante* of agent 2, β) together with relatively low level of the prize value V, or alternatively that the prize value V is relatively high within its range of the admissible values. In turn, the extra conditions in C3 for active agent 2 together with inactive agent 1 require that either the intensity of affirmative action policy is low enough (lower than the marginal cost of competitive effort *ex ante* of agent 2, β) together with relatively low level of the prize value V, or alternatively that the the prize value V is relatively high within the range of its admissible values.

3.3 Marginal Cost of Effort Levels Under Equal Treatment Policy

It is interesting to notice, that even if the AA policy is not implemented (and the ET option is in place then), the agents may have incentives to invest in their learning effort and improve their original levels of marginal cost of competitive effort *ex ante*, β_{0i}. Setting $\alpha = 1$ in eq. (12).and we obtain that under the equal treatment policy the equilibrium levels of the marginal cost of competitive effort of agents are

$$(\beta_{11}^{*}, \beta_{12}^{*}) = \begin{cases} \left(\frac{\left(\gamma_1+\beta^2\gamma_2\right)^{3/2}}{\sqrt{V}\beta^2\gamma_2}, \frac{\left(\gamma_1+\beta^2\gamma_2\right)^{3/2}}{\sqrt{V}\gamma_1} \right), \text{if} & C1, \\ \left(\frac{\beta\gamma_1^{1/3}}{\sqrt{\left(\left(V\beta^2\right)^{1/3}-\left(\gamma_1\right)^{1/3}\right)^2}}, \beta \right), \text{ if} & C2, \\ \left(1, \frac{\left(\beta^2\gamma_2\right)^{1/3}}{\sqrt{\left(V^{1/3}-\left(\beta^2\gamma_2\right)^{1/3}\right)^2}} \right), \text{ if} & C3, \\ (1, \beta), & \text{otherwise.} \end{cases} \tag{13}$$

As we see, for all active agents they are different from their original levels of 1 and β, respectively for agent 1 and 2. As changing the levels of marginal cost of competitive effort requires learning effort, this shows that even if there is no AA in place, some investment in learning effort may occur.

3.4 Effects of Affirmative Action on Marginal Cost of Competitive Effort and Learning Effort Levels

Agents' investment in learning effort in stage 0 of the game affects the marginal cost of competitive effort in stage 1. In this part of our analysis we discuss the effects of implementation the various policy options on the learning effort levels of players and the resulting levels of marginal cost of competitive effort. We will consider the agents' behavior under affirmative action policy of various intensities ($\alpha > 1$) and equal treatment policy ($\alpha = 1$). In the following the equal treatment policy option is treated as a special case of the affirmative action policy option when its intensity is zero.

3.4.1 Agent 1

Agent 1 is the non-discriminated player ($\beta_{0i} = 1$) and although they are not the object of the AA policy, their marginal cost of competitive effort is affected. The following proposition characterizes how the AA policy of various intensity levels (including the ET policy as its special case) affects the equilibrium level of the marginal cost of competitive effort of that player.

Proposition 1 *The equilibrium level of the marginal cost of competitive effort of agent 1, β_{11} is*

(a) when both agents are active (C1 holds):

(a.i) for when $\frac{\gamma_1}{\gamma_2} < \frac{\beta^2}{2}$ holds:

- decreasing in α, if

$$1 \leq \alpha < \beta\sqrt{\frac{\gamma_2}{2\gamma_1}},$$

- increasing in α*, if*

$$\alpha > \beta\sqrt{\frac{\gamma_2}{2\gamma_1}},$$

- has a minimum in α*, if*

$$\alpha = \beta\sqrt{\frac{\gamma_2}{2\gamma_1}}.$$

(a.ii) for when $\frac{\gamma_1}{\gamma_2} = \frac{\beta^2}{2}$ *holds:*

- increasing in α*, if*

$$\alpha > 1,$$

- has a minimum in α*, if*

$$\alpha = 1,$$

(a.iii) for when $\frac{\gamma_1}{\gamma_2} > \frac{\beta^2}{2}$ *holds:*

- increasing in α*, if*

$$\alpha \geq 1,$$

b) when agent 1 only is active (C2 holds)

(b.i) for when $V < \frac{\gamma_1}{\beta^2}$ *holds:*

- decreasing in α*, if*

$$\alpha \geq 1,$$

(b.ii) for when $V = \frac{\gamma_1}{\beta^2}$ *holds:*

- decreasing in α*, if*

$$\alpha > 1,$$

(b.iii) for when $\frac{\gamma_1}{\beta^2} < V < \frac{27\gamma_1}{\beta^2}$ *holds:*

- decreasing in α*, if*

$$\alpha > \beta\sqrt{\frac{V}{\gamma_1}},$$

- increasing in α*, if*

$$1 \leq \alpha < \beta\sqrt{\frac{V}{\gamma_1}},$$

(b.iv) for when $V = \frac{27\gamma_1}{\beta^2}$ *holds:*

- decreasing in α*, if*

$$\alpha > \beta\sqrt{\frac{V}{\gamma_1}},$$

- increasing in α*, if*

$$1 < \alpha < \beta\sqrt{\frac{V}{\gamma_1}},$$

- has a minimum in α*, if*

$$\alpha = 1,$$

(b.v) for when $V > \frac{27\gamma_1}{\beta^2}$ *holds:*

- *decreasing in* α, *if*

$$1 \le \alpha < \beta\sqrt{\tfrac{V}{27\gamma_1}} \cup \alpha > \beta\sqrt{\tfrac{V}{\gamma_1}}$$

- *increasing in* α, *if*

$$\beta\sqrt{\tfrac{V}{27\gamma_1}} < \alpha < \beta\sqrt{\tfrac{V}{\gamma_1}}$$

- *has a minimum in* α, *if*

$$\alpha = \beta\sqrt{\tfrac{V}{27\gamma_1}}.$$

Proof. For part a) of the Proposition 1 (when C1 holds and both agents are active), using the corresponding equilibrium marginal effort levels of agent 1 in (12) and differentiating with respect to α yields

$$\frac{\left(2\alpha^2\gamma_1-\beta^2\gamma_2\right)\sqrt{\alpha^2\gamma_1+\beta^2\gamma_2}}{\sqrt{V}\alpha^2\beta^2\gamma_2},$$

which is negative if

$$\tfrac{\gamma_1}{\gamma_2} < \tfrac{\beta^2}{2} \cap \alpha < \beta\sqrt{\tfrac{\gamma_2}{2\gamma_1}} \cap \alpha \ge 1,$$

positive if

$$\left(\tfrac{\gamma_1}{\gamma_2} < \tfrac{\beta^2}{2} \cap \alpha > \beta\sqrt{\tfrac{\gamma_2}{2\gamma_1}}\right) \cup \left(\tfrac{\gamma_1}{\gamma_2} > \tfrac{\beta^2}{2} \cap \alpha \ge 1\right) \cup \left(\tfrac{\gamma_1}{\gamma_2} = \tfrac{\beta^2}{2} \cap \alpha > 1\right),$$

and equal to zero, if

$$\left(\tfrac{\gamma_1}{\gamma_2} < \tfrac{\beta^2}{2} \cap \alpha = \beta\sqrt{\tfrac{\gamma_2}{2\gamma_1}}\right) \cup \left(\tfrac{\gamma_1}{\gamma_2} = \tfrac{\beta^2}{2} \cap \alpha = 1\right).$$

This can be rewritten as in the Proposition, proving its part a).

For part b) of the Proposition (when C2 holds and agent 1 only is active), using the corresponding equilibrium marginal effort levels of agent 1 in (12) and differentiating with respect to α yields

$$\frac{3\beta(\alpha\gamma_1)^{2/3}-\left(V\beta^5\gamma_1\right)^{1/3}}{3\alpha^{4/3}\left(\left(V\beta^2\right)^{1/3}-(\alpha^2\gamma_1)^{1/3}\right)\sqrt{\left(\left(V\beta^2\right)^{1/3}-(\alpha^2\gamma_1)^{1/3}\right)^2}},$$

which is negative if

$$\begin{aligned}
&\left(V < \tfrac{\gamma_1}{\beta^2} \cap \alpha \ge 1\right) \cup \\
&\left(V = \tfrac{\gamma_1}{\beta^2} \cap \alpha > 1\right) \cup \\
&\left(V > \tfrac{\gamma_1}{\beta^2} \cap \alpha > \beta\sqrt{\tfrac{V}{\gamma_1}}\right) \cup
\end{aligned}$$

$$\left(V > \tfrac{27\gamma_1}{\beta^2} \cap \alpha < \beta\sqrt{\tfrac{V}{27\gamma_1}} \cap \alpha \geq 1\right),$$

positive if

$$\alpha < \beta\sqrt{\tfrac{V}{\gamma_1}} \cap \left(\begin{array}{c} \left(V = \tfrac{27\gamma_1}{\beta^2} \cap \alpha > 1\right) \cup \left(\tfrac{\gamma_1}{\beta^2} < V < \tfrac{27\gamma_1}{\beta^2} \cap \alpha \geq 1\right) \cup \\ \left(V > \tfrac{27\gamma_1}{\beta^2} \cap \alpha > \beta\sqrt{\tfrac{V}{27\gamma_1}}\right) \end{array} \right)$$

and equal to zero, if

$$\left(V > \tfrac{27\gamma_1}{\beta^2} \cap \alpha = \beta\sqrt{\tfrac{V}{27\gamma_1}}\right) \cup \left(V = \tfrac{27\gamma_1}{\beta^2} \cap \alpha = 1\right).$$

This can be rewritten as in the Proposition, proving its part b). ■

It follows from this Proposition 1 that the marginal cost of competitive effort of agent 1, β_{11} changes as the intensity of the *AA* policy varies. For when both agents, 1 and 2, are active the behavior of β_{11} is dependent on the relation between $\frac{\gamma_1}{\gamma_2}$ and $\frac{\beta^2}{2}$. If $\frac{\gamma_1}{\gamma_2} \leq \frac{\beta^2}{2}$ holds, that is when the level of discrimination of agent 2 *ex ante* β is high and/or when the marginal cost of learning effort of agent 1, γ_1 is relatively low as compared to the marginal cost of learning effort of agent 2, γ_2 (learning is cheap for agent 1 in relative terms), then β_{11} is first decreasing in α up to $\alpha = \beta\sqrt{\frac{\gamma_2}{2\gamma_1}}$, where it reaches its minimum and then it starts to increase. By eq. (2), this means that initially, as the response to the increase in the intensity of the affirmative action policy, agent 1 invests more and more in learning effort to improve their marginal cost of competitive effort. This investment reaches its maximum when $\alpha = \beta\sqrt{\frac{\gamma_2}{2\gamma_1}}$. After that point the incentives to invest further are somehow reduced and the investment starts to drop. In turn, if $\frac{\gamma_1}{\gamma_2} > \frac{\beta^2}{2}$holds, that is when the level of discrimination of agent 2 *ex ante* β is low and/or when the marginal cost of learning effort of agent 1, γ_1 is relatively high as compared to the marginal cost of learning effort of agent 2, γ_2 (learning is costly for agent 1 in relative terms), then β_{11} is always increasing. This means that, as the response to the increase of the intensity of the affirmative action policy, agent 1 invests less and less in learning to improve their marginal cost of competitive effort.

A more complicated picture emerges when agent 1 is the only active agent and condition C2 holds. In this case, the behavior of β_{11} as a function of α is dependent on the relation between V, $\frac{\gamma_1}{\beta^2}$ and $\frac{27\gamma_1}{\beta^2}$ and is additionally affected by the existence of the discontinuity point at $\alpha = \beta\sqrt{\frac{V}{\gamma_1}}$[9]. It is worth noting here that within the space defined

[9] This discontinuity point at $\alpha = \beta\sqrt{\frac{V}{\gamma_1}}$ doesn't belong to the set defined by C2 and lies in between the parameter space areas defined by $\left(\alpha > \beta \cap V < \frac{(\alpha-\beta)^3\gamma_1}{\alpha\beta^2}\right)$ and $\left(V > \frac{(\alpha+\beta)^3\gamma_1}{\alpha\beta^2}\right)$. Closer investigation reveals that in the neighbourhood of $\alpha = \beta\sqrt{\frac{V}{\gamma_1}}$ the learning effort level of agent 1 is zero. This means that their level of marginal cost of competitive effort β_{11} is constant in this range of α and always equal to its level ex ante of 1.

by C2 whenever $\alpha > \beta\sqrt{\frac{V}{\gamma_1}}$, then independently of the relation between V, $\frac{\gamma_1}{\beta^2}$ and $\frac{27\gamma_1}{\beta^2}$, β_{11} is always decreasing in α. If $V < \frac{\gamma_1}{\beta^2}$holds, that is when the level of discrimination of agent 2 *ex ante* β is low and/or when the marginal cost of learning effort of agent 1, γ_1 is high (learning is costly for agent 1), then always $\alpha > \beta\sqrt{\frac{V}{\gamma_1}}$, and by the previous observation β_{11} is always decreasing in α for all $\alpha \geq 1$. This means that in this case the increase in the intensity of the affirmative action policy always results in agent 1 investing more and more in learning to improve their marginal cost of competitive effort. If $V = \frac{\gamma_1}{\beta^2}$, then there is the discontinuity point is at $\alpha = 1$, and β_{11} is always decreasing in α, for all $\alpha > 1$. In turn, if $\frac{\gamma_1}{\beta^2} < V < \frac{27\gamma_1}{\beta^2}$ holds, then β_{11} is initially increasing up to the discontinuity point for all $\alpha < \beta\sqrt{\frac{V}{\gamma_1}}$, and then decreasing for all $\alpha > \beta\sqrt{\frac{V}{\gamma_1}}$[10]. This means that in this case, as the response to the increase in the intensity of the affirmative action policy, for small intensity levels of the AA policy, agent 1 invests less and less in their learning effort to improve marginal cost of competitive effort and later when the AA intensity level crosses the neighborhood of the discontinuity point, this investment starts to go up. Finally, if $V \geq \frac{27\gamma_1}{\beta^2}$, that is when the level of discrimination of agent 2 *ex ante* β is high and/or when the marginal cost of learning effort of agent 1, γ_1 is low (learning is cheap for agent 1), then β_{11} is first decreasing in α up to $\alpha = \beta\sqrt{\frac{V}{27\gamma_1}}$, where it reaches its minimum, and then it increases for all $\alpha < \beta\sqrt{\frac{V}{\gamma_1}}$, and decreases for all $\alpha > \beta\sqrt{\frac{V}{\gamma_1}}$[11]. This means that initially for small intensity levels of the AA policy, as the response to the increase in the intensity of the affirmative action policy, agent 1 invests more and more effort in their learning to improve their marginal cost of competitive effort. This investment reaches its maximum when $\alpha = \beta\sqrt{\frac{V}{27\gamma_1}}$. After that point the incentives to invest further in education are reduced and the investment starts to drop. However, this happens only up to the neighborhood of the discontinuity point, and when the AA intensity level crosses this point, this investment starts to go up again.

As there is inverse relation between the marginal cost of competitive effort in stage 1 and the level of learning effort exerted as given in eq. (2), using Proposition 1 we can easily determine the level of the intensity of the AA policy, including the ET option, that induces for agent 1 the highest level of the learning effort. The following Conclusion summarizes the results regarding this problem:

Conclusion 1 *The equilibrium level of the learning effort of agent 1, ϵ_1^*, is maximized in α*

[10] As we noted earlier, in the neighbourhood of the discontinuity point the level of marginal cost of competitive effort β_{11} is constant in α and equal to its level ex ante of 1.

[11] As earlier, in the neighbourhood of the discontinuity point β_{11} is constant in α and equal to 1.

a) when both agents are active (C1 holds):

$$\begin{array}{c} \textit{globally at } \alpha = \beta\sqrt{\frac{\gamma_2}{2\gamma_1}}, \textit{ if } \frac{\gamma_1}{\gamma_2} \leq \frac{\beta^2}{2}, \\ \textit{globally at } \alpha = 1, \quad \textit{if } \frac{\gamma_1}{\gamma_2} > \frac{\beta^2}{2}, \end{array}$$

b) when agent 1 only is active (C2 holds):

$$\begin{array}{cc} \textit{globally at } \alpha \to \infty, & \textit{if} \quad V \leq \frac{\gamma_1}{\beta^2} \\ \begin{cases} \textit{locally at } \alpha = 1, \\ \textit{globally at } \alpha \to \infty, \end{cases} & \textit{if } \frac{\gamma_1}{\beta^2} < V < \frac{27\gamma_1}{\beta^2} \\ \begin{cases} \textit{locally at } \alpha = \beta\sqrt{\frac{V}{27\gamma_1}}, \\ \textit{globally at } \alpha \to \infty, \end{cases} & \textit{if} \quad V \geq \frac{27\gamma_1}{\beta^2}. \end{array}$$

Proof. To prove the Conclusion we will use the fact that there is inverse relation between the marginal cost of competitive effort in stage 1 and the level of learning effort exerted as given in eq. (2).

For part a) of the Conclusion, when both agents are active (C1 holds): using the results summarized in Proposition 1 we obtain that the marginal cost of the competitive effort of player 1 is minimized in α

$$\begin{array}{c} \text{globally at } \alpha = \beta\sqrt{\frac{\gamma_2}{2\gamma_1}}, \text{ if } \frac{\gamma_1}{\gamma_2} \leq \frac{\beta^2}{2}, \\ \text{globally at } \alpha = 1, \quad \text{if } \frac{\gamma_1}{\gamma_2} > \frac{\beta^2}{2}. \end{array}$$

By eq. (2), the same conditions define where the level of the learning effort of player 1 is at maximum, proving part a) of the Conclusion.

For part b) of the Conclusion, when agent 1 only is active (C2 holds): using the results summarized in Proposition 1 and the fact that limit of the marginal cost of competitive effort of player 1 when $\alpha \to \infty$ is zero, we obtain that the marginal cost of the competitive effort of player 1 is minimized in α:

$$\begin{array}{cc} \text{globally at } \alpha \to \infty, & \text{if} \quad V \leq \frac{\gamma_1}{\beta^2} \\ \begin{cases} \text{locally at } \alpha = 1, \\ \text{globally at } \alpha \to \infty, \end{cases} & \text{if } \frac{\gamma_1}{\beta^2} < V < \frac{27\gamma_1}{\beta^2} \\ \begin{cases} \text{locally at } \alpha = \beta\sqrt{\frac{V}{27\gamma_1}}, \\ \text{globally at } \alpha \to \infty, \end{cases} & \text{if} \quad V \geq \frac{27\gamma_1}{\beta^2}. \end{array}$$

By eq. (2), the same conditions define where the level of the learning effort of player 1 is at maximum. As player 1 is the only one who exerts effort in this case, the same conditions define where the maximum of the total level of equilibrium learning effort is, which proves part b) of the Conclusion. ■

It is interesting to consider Conclusion 1 in terms of the policy options that induce the highest level of learning effort of agent 1. If both agents are active players, then there is a well defined global maximum of the learning effort level, that depends on the relation between $\frac{\gamma_1}{\gamma_2}$ and $\frac{\beta^2}{2}$. If $\frac{\gamma_1}{\gamma_2} \leq \frac{\beta^2}{2}$ holds (the level of discrimination of agent 2 *ex ante* β is high and/or when the marginal cost of learning effort of agent 1, γ_1 is relatively low as compared to the marginal cost of learning effort of agent 2, γ_2), then the maximum learning effort level is exerted when the implemented policy is AA and its intensity is given by $\alpha = \beta\sqrt{\frac{\gamma_2}{2\gamma_1}}$. In turn, if $\frac{\gamma_1}{\gamma_2} > \frac{\beta^2}{2}$ holds (the level of discrimination of agent 2 *ex ante* β is low and/or when the marginal cost of learning effort of agent 1, γ_1 is relatively high as compared to the marginal cost of learning effort of agent 2, γ_2), then the policy that induces the highest level of learning effort for agent 1 is the ET option.

When agent 1 only is active, the situation is much more complex. Depending on the specific combination of the model parameters, there can be either a (proper) local maximum with no (proper) global maximum(a), or no (proper) global maximum at all. For $V \leq \frac{\gamma_1}{\beta^2}$, there is neither (proper) global maximum nor local maximum, and the higher the intensity of the AA policy, the higher the learning effort level of player 1. If $V > \frac{\gamma_1}{\beta^2}$, then again there is no (proper) global maximum, but there is a local maximum. Again, (globally) the higher the intensity of the AA policy, the higher the learning effort level of player 1. Additionally however, there is a local maximum at $\alpha = 1$ for $\frac{\gamma_1}{\beta^2} < V < \frac{27\gamma_1}{\beta^2}$, and $\alpha = \beta\sqrt{\frac{V}{27\gamma_1}}$ for $V \geq \frac{27\gamma_1}{\beta^2}$. This means that in the former case locally the ET option induces the highest learning effort level, and in the latter (again locally) the highest learning effort level is exerted when the implemented policy is the AA and its intensity is given by $\alpha = \beta\sqrt{\frac{V}{27\gamma_1}}$.

3.4.2 Agent 2

Agent 2 is the discriminated player ($\beta_{0i} = \beta$) and they are the object of the AA policy. The following proposition characterizes how the AA policy of various intensity levels (including the ET policy as its special case) affects the equilibrium level of the marginal cost of competitive effort of this player:

Proposition 2 *The equilibrium level of the marginal cost of competitive effort of agent 2, β_{12}, is:*

a) when both agents are active (C1 holds):

(a.i) for when $\frac{\gamma_1}{\gamma_2} < 2\beta^2$ holds:

- decreasing in α, if

$$1 \leq \alpha < \beta\sqrt{\frac{2\gamma_2}{\gamma_1}},$$

- increasing in α, if

$$\alpha > \beta\sqrt{\frac{2\gamma_2}{\gamma_1}},$$

- has a minimum in α, if

$$\alpha = \beta\sqrt{\frac{2\gamma_2}{\gamma_1}}.$$

(a.ii) for when $\frac{\gamma_1}{\gamma_2} = 2\beta^2$ *holds:*

- increasing in α*, if*

$$\alpha > 1,$$

- has a minimum in α*, if*

$$\alpha = 1,$$

(a.iii) for when $\frac{\gamma_1}{\gamma_2} > 2\beta^2$ *holds:*

- increasing in α*, if*

$$\alpha \geq 1,$$

b) when agent 2 only is active (C3 holds)

(b.i) for when $V < \beta^2\gamma_2$ *holds:*

- decreasing in α*, if*

$$\beta\sqrt{\tfrac{\gamma_2}{V}} < \alpha < \beta\sqrt{\tfrac{27\gamma_2}{V}}$$

- increasing in α*, if*

$$\left(1 \leq \alpha < \beta\sqrt{\tfrac{\gamma_2}{V}}\right) \cup \alpha > \beta\sqrt{\tfrac{27\gamma_2}{V}}$$

- has a minimum in α*, if*

$$\alpha = \beta\sqrt{\tfrac{27\gamma_2}{V}},$$

(b.ii) for when $V = \beta^2\gamma_2$ *holds:*

- decreasing in α*, if*

$$1 < \alpha < \beta\sqrt{\tfrac{27\gamma_2}{V}},$$

- increasing in α*, if*

$$\alpha > \beta\sqrt{\tfrac{27\gamma_2}{V}}$$

- has a minimum in α*, if*

$$\alpha = \beta\sqrt{\tfrac{27\gamma_2}{V}},$$

(b.iii) for when $\beta^2\gamma_2 < V < 27\beta^2\gamma_2$ *holds:*

- decreasing in α*, if*

$$1 \leq \alpha < \beta\sqrt{\tfrac{27\gamma_2}{V}},$$

- increasing in α*, if*

$$\alpha > \beta\sqrt{\tfrac{27\gamma_2}{V}},$$

- has a minimum in α*, if*

$$\alpha = \beta\sqrt{\tfrac{27\gamma_2}{V}},$$

(b.iv) for when $V = 27\beta^2\gamma_2$ holds:
- *increasing in α, if*

$$\alpha > 1,$$

- *has a minimum in α, if*

$$\alpha = 1,$$

(b.v) for when $V > 27\beta^2\gamma_2$ holds:
- *increasing in α, if*

$$\alpha \geq 1.$$

Proof. For part a) of the Proposition 2 (when C1 holds and both agents are active), using the corresponding equilibrium marginal effort levels of agent 2 in (12) and differentiating with respect to α yields

$$\frac{\left(\alpha^2\gamma_1 - 2\beta^2\gamma_2\right)\sqrt{\alpha^2\gamma_1+\beta^2\gamma_2}}{\sqrt{V}\alpha^3\gamma_1},$$

which is negative if

$$\tfrac{\gamma_1}{\gamma_2} < 2\beta^2 \cap \alpha < \beta\sqrt{\tfrac{2\gamma_2}{\gamma_1}} \cap \alpha \geq 1,$$

positive if

$$\left(\tfrac{\gamma_1}{\gamma_2} < 2\beta^2 \cap \alpha > \beta\sqrt{\tfrac{2\gamma_2}{\gamma_1}}\right) \cup \left(\tfrac{\gamma_1}{\gamma_2} > 2\beta^2 \cap \alpha \geq 1\right) \cup \left(\tfrac{\gamma_1}{\gamma_2} = 2\beta^2 \cap \alpha > 1\right),$$

and equal to zero, if

$$\left(\tfrac{\gamma_1}{\gamma_2} < 2\beta^2 \cap \alpha = \beta\sqrt{\tfrac{2\gamma_2}{\gamma_1}}\right) \cup \left(\tfrac{\gamma_1}{\gamma_2} = 2\beta^2 \cap \alpha = 1\right).$$

This can be rewritten as in the Proposition, proving its part a).

For part b) of the Proposition (when C3 holds and agent 2 only is active), using the corresponding equilibrium marginal effort levels of agent 2 in (12) and differentiating with respect to α yields

$$\frac{\left(V\alpha^2\gamma_2\right)^{1/3}\beta - 3\left(\beta^5\gamma_2^2\right)^{1/3}}{3\left((V\alpha^2\beta)^{1/3} - \beta\gamma_2^{1/3}\right)\sqrt{\left((V\alpha^2)^{1/3} - \left(\beta^2\gamma_2\right)^{1/3}\right)^2}},$$

which is negative if

$$\alpha < \beta\sqrt{\tfrac{27\gamma_2}{V}} \cap \left(\begin{array}{c}\left(V < \beta^2\gamma_2 \cap \alpha > \beta\sqrt{\tfrac{\gamma_2}{V}}\right) \cup \left(V = \beta^2\gamma_2 \cap \alpha > 1\right) \cup \\ \left(\beta^2\gamma_2 < V < 27\beta^2\gamma_2 \cap \alpha \geq 1\right)\end{array}\right),$$

positive if

$$\left(V < \beta^2\gamma_2 \cap \alpha < \beta\sqrt{\tfrac{\gamma_2}{V}}\right) \cup \left(V < 27\beta^2\gamma_2 \cap \alpha > \beta\sqrt{\tfrac{27\gamma_2}{V}}\right) \cup$$
$$\left(V = 27\beta^2\gamma_2 \cap \alpha > 1\right) \cup \left(V > 27\beta^2\gamma_2 \cap \alpha \geq 1\right),$$

and equal to zero, if

$$\left(V < 27\beta^2\gamma_2 \cap \alpha = \beta\sqrt{\frac{27\gamma_2}{V}}\right) \cup \left(V = 27\beta^2\gamma_2 \cap \alpha = 1\right).$$

This can be rewritten as in the Proposition, proving its part b). ■

It follows from this Proposition 2 that the marginal cost of competitive effort of agent 2, β_{12} changes as the intensity of the *AA* policy varies. For when both agents, 1 and 2, are active the behavior of β_{12} is dependent on the relation between $\frac{\gamma_1}{\gamma_2}$ and $2\beta^2$. If $\frac{\gamma_1}{\gamma_2} \leq 2\beta^2$ holds, that is when the level of discrimination of agent 2 *ex ante* β is high and/or when the marginal cost of learning effort of agent 2, γ_2 is relatively high as compared to the marginal cost of learning effort of agent 1, γ_1 (learning is costly for agent 2 in relative terms), then β_{12} is first decreasing in α up to $\alpha = \beta\sqrt{\frac{2\gamma_2}{\gamma_1}}$, where it reaches its minimum and then it starts to increase. This means that initially, as the response to the increase in the intensity of the affirmative action policy, agent 2 invests more and more effort in their learning to improve their marginal cost of competitive effort. This investment reaches its maximum when $\alpha = \beta\sqrt{\frac{2\gamma_2}{\gamma_1}}$. After that point the incentives to invest further in learning effort become lower and the investment drops. In turn, if $\frac{\gamma_1}{\gamma_2} > 2\beta^2$ holds, that is when the level of discrimination of agent 2 *ex ante* β is low and/or when the marginal cost of learning effort of agent 2, γ_2 is relatively low as compared to the marginal cost of learning effort of agent 1, γ_2 (learning is cheap for agent 2 in relative terms), then β_{12} is always increasing. This means that, as the response to the increase in the intensity of the affirmative action policy, agent 2 invests less and less in their learning effort.

The relation between the AA intensity level α and the the marginal cost of competitive effort of agent 2 β_{12} is more complex when agent 2 is the only active agent. Then, the behavior of β_{12} as a function of α is dependent not only on the relation between V, $\beta^2\gamma_2$ and $27\beta^2\gamma_2$ but is also affected by the existence of the discontinuity point at $\alpha = \beta\sqrt{\frac{\gamma_2}{V}}$[12]. It is important to note here that within the space defined by C3, if $\alpha < \beta\sqrt{\frac{\gamma_2}{V}}$, then independently of the relation between V, $\beta^2\gamma_2$ and $27\beta^2\gamma$, β_{12} is always decreasing in α. If $V \leq \beta^2\gamma_2$ holds, that is when the level of discrimination of agent 2 *ex ante* β is high and/or when the marginal cost of learning effort of agent 2, γ_2 is high (learning is costly for agent 2), then β_{12} is initially increasing up to the discontinuity point for all $\alpha < \beta\sqrt{\frac{\gamma_2}{V}}$, and then decreasing for $\alpha > \beta\sqrt{\frac{\gamma_2}{V}}$[13] until the minimum

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[12]This discontinuity point at $\alpha = \beta\sqrt{\frac{\gamma_2}{V}}$ doesn't belong to the set defined by C3 and lies in between the parameter space areas defined by $\left(\alpha < \beta \cap V < \frac{(\beta-\alpha)^3\gamma_2}{\alpha^2\beta}\right)$ and $\left(V > \frac{(\alpha+\beta)^3\gamma_2}{\alpha^2\beta}\right)$. Closer investigation reveals that in the neighbourhood of $\alpha = \beta\sqrt{\frac{\gamma_2}{V}}$ the learning effort level of agent 2 is zero. This means that their level of marginal cost of competitive effort β_{12} is constant in this range of α and always equal to its level ex ante of β.

[13]As noted earlier, in the neighbourhood of the discontinuity point β_{12} is constant in α and equal to β.

$\alpha = \beta\sqrt{\frac{27\gamma_2}{V}}$ is reached. After that point, β_{12} starts to increase again. This means that in this case, as the response to the increase in the intensity of the affirmative action policy, for its small intensity levels, agent 2 invests less and less in their learning effort and later when the AA intensity level crosses the neighborhood of the discontinuity point, this investment starts to go up and reaches its maximum when $\alpha = \beta\sqrt{\frac{27\gamma_2}{V}}$. After that point the incentives to invest in education are lower and the learning effort investment levels become smaller and smaller, as α increases. If $\beta^2\gamma_2 < V \leq 27\beta^2\gamma_2$, then the evolution of β_{12} is very similar to the one for $V \leq \beta^2\gamma_2$, except for the fact that in this case there is no discontinuity point in α. That is β_{12} is first decreasing and later increasing with the minimum at $\alpha = \beta\sqrt{\frac{27\gamma_2}{V}}$. This means that initially, as the response to the increase in the intensity of the affirmative action policy, agent 2 invests more and more in their learning effort to improve their marginal cost of competitive effort. This investment reaches its maximum at $\alpha = \beta\sqrt{\frac{27\gamma_2}{V}}$. After that point the incentives to invest in learning effort are reduced and the investment starts to drop. In turn, if $V > 27\beta^2\gamma_2$ holds, that is when the level of discrimination of agent 2 *ex ante* β is low and/or when the marginal cost of learning effort of agent 2, γ_2 is low (learning is cheap for agent 2), then β_{12} is always increasing. This means that, as the response to the increase in the intensity of the affirmative action policy, agent 2 invests less and less in their learning effort to improve their marginal cost of competitive effort.

As for agent 1, we study also the levels of the intensity of the AA policy, including the ET option, that induces for agent 2 the highest level of the learning effort. Our results regarding this problem are summarized in the following Conclusion:

Conclusion 2 *The equilibrium level of the learning effort of agent 2, ϵ_2^*, is maximized in α*

a) when both agents are active (C1 holds):

$$\begin{array}{c} \textit{globally at } \alpha = \beta\sqrt{\frac{2\gamma_2}{\gamma_1}},\ \textit{if } \frac{\gamma_1}{\gamma_2} \leq 2\beta^2, \\ \textit{globally at } \alpha = 1, \quad \textit{if } \frac{\gamma_1}{\gamma_2} > 2\beta^2, \end{array}$$

b) when agent 2 only is active (C3 holds):

$$\begin{array}{cc} \left\{\begin{array}{c} \textit{locally at } \alpha = \beta\sqrt{\frac{V}{27\gamma_1}}, \\ \textit{globally at } \alpha = 1 \end{array}\right. & \textit{if} \qquad V \leq \beta^2\gamma_2, \\ \textit{globally at } \alpha = \beta\sqrt{\frac{V}{27\gamma_1}}, & \textit{if } \beta^2\gamma_2 < V \leq 27\beta^2\gamma_2 \\ \textit{globally at } \alpha = 1, & \textit{if} \qquad V > 27\beta^2\gamma_2. \end{array}$$

Proof. As for Conclusion 1, to prove Conclusion 2 we will use the fact that there is inverse relation between the marginal cost of competitive effort in stage 1 and the level of learning effort exerted as given in eq. (2).

For part a) of the Conclusion, when both agents are active (C1 holds): using the results summarized in Proposition 2 we obtain that the marginal cost of the competitive effort of player 2 is minimized in α

$$\begin{array}{r} \text{globally at } \alpha = \beta\sqrt{\frac{2\gamma_2}{\gamma_1}}, \text{if } \frac{\gamma_1}{\gamma_2} \leq 2\beta^2, \\ \text{globally at } \alpha = 1, \quad \text{if } \frac{\gamma_1}{\gamma_2} > 2\beta^2, \end{array}$$

By eq. (2), the same conditions define where the level of the learning effort of player 2 is at maximum, proving part a) of the Conclusion.

For part a) of the Conclusion, when agent 2 only is active (C3 holds), we use the results summarized in Proposition 2 and the relation between the value of the marginal cost of competitive effort of player 2 when $\alpha = 1$, $\beta_{12}(1)$ and at a local minimum $\alpha = \beta\sqrt{\frac{V}{27\gamma_1}}$, $\beta_{12}\left(\beta\sqrt{\frac{V}{27\gamma_1}}\right)$. Using some algebra within the domain of the admissible values of the model parameters it can be shown that

$$\begin{array}{l} \beta_{12}(1) < \beta_{12}\left(\beta\sqrt{\frac{V}{27\gamma_1}}\right), \text{if} \qquad V \leq \beta^2\gamma_2, \\ \beta_{12}(1) > \beta_{12}\left(\beta\sqrt{\frac{V}{27\gamma_1}}\right), \text{if } \beta^2\gamma_2 < V < 27\beta^2\gamma_2, \\ \beta_{12}(1) = \beta_{12}\left(\beta\sqrt{\frac{V}{27\gamma_1}}\right), \text{if} \qquad V = 27\beta^2\gamma_2. \end{array}$$

Using this result and Proposition 2 we obtain that the marginal cost of the competitive effort of player 2 is minimized in α

$$\begin{array}{cl} \left\{\begin{array}{c} \text{locally at } \alpha = \beta\sqrt{\frac{V}{27\gamma_1}}, \\ \text{globally at } \alpha = 1 \end{array}\right. & \text{if} \qquad V \leq \beta^2\gamma_2, \\ \text{globally at } \alpha = \beta\sqrt{\frac{V}{27\gamma_1}}, & \text{if} \, \beta^2\gamma_2 < V \leq 27\beta^2\gamma_2, \\ \text{globally at } \alpha = 1, & \text{if} \qquad V > 27\beta^2\gamma_2. \end{array}$$

By eq. (2), the same conditions define where the level of the learning effort of player 2 is at maximum. As player 2 is the only one who exerts effort in this case, the same conditions define where the maximum of the total level of equilibrium learning effort is, which proves part b) of the Conclusion. ■

We can interpret the results in Conclusion 2 in terms of the policy options inducing the highest level of learning effort of agent 2. If both agents are active players, the there is a well defined global maximum of the learning effort level, that depends on the relation between $\frac{\gamma_1}{\gamma_2}$ and $2\beta^2$. If $\frac{\gamma_1}{\gamma_2} \leq 2\beta^2$ holds (the level of discrimination of agent 2 *ex ante* β is high and/or when the marginal cost of learning effort of agent 1, γ_1 is relatively low as compared to the marginal cost of learning effort of agent 2, γ_2), then the maximum leaning effort level is exerted when the implemented policy is AA of the intensity level given by $\alpha = \beta\sqrt{\frac{2\gamma_2}{\gamma_1}}$. In turn, if $\frac{\gamma_1}{\gamma_2} > 2\beta^2$ holds (the level of discrimination of agent 2 *ex ante* β is low and/or when the marginal cost of learning effort of agent 1, γ_1 is relatively high as compared to the marginal cost of learning effort of agent 2, γ_2), then the policy that induces the highest level of learning effort for agent 2 is the ET option.

When agent 2 only is active, then depending on the specific combinations of the model parameters, there is either one (proper) global maximum, or both a (proper) global maximum and a (proper) local maximum. For $V > \beta^2\gamma_2$ there is a global maximum only, and this maximum is at $\alpha = \beta\sqrt{\frac{V}{27\gamma_1}}$ for $\beta^2\gamma_2 < V \leq 27\beta^2\gamma_2$, and at $\alpha = 1$ for $V > 27\beta^2\gamma_2$. This means that in the latter case this is the ET option that induces the highest learning effort level of player 2, and in the former this is the AA policy of the intensity level given by $\alpha = \beta\sqrt{\frac{V}{27\gamma_1}}$. If $V \leq \beta^2\gamma_2$, then there is both a (proper) global maximum and a (proper) local maximum. Again, there is a maximum at $\alpha = \beta\sqrt{\frac{V}{27\gamma_1}}$, but now it is local only. Apart from that there is a global maximum at $\alpha = 1$. This means that in this case globally the ET option induces the highest learning effort level of player 2.

3.4.3 Agent 1 vs Agent 2

As a result of exerting learning effort in stage 0 of the game, the values of the parameters of marginal cost of competitive effort are reduced. The size of this reduction is different for different agents and depends on specific combinations of the values of the model parameters (including the intensity of the AA policy, α). Given this, we might expect that for some constellations of the model parameters, this reduction for the discriminated player - agent 2 is much higher than for the non-discriminated player - agent 1, potentially leading to a situation in which the *ex ante* weaker player actually becomes the stronger one in the final stage of the game. In this section we investigate this potential phenomenon and study the conditions for it to take place. Our results regarding this issue are summarized in the following Proposition:

Proposition 3 *In equilibrium, the level of the marginal cost of competitive effort of agent 1 becomes higher than the level of the marginal cost of competitive effort of agent 2 ($\beta^*_{11} > \beta^*_{12}$) when the following relations hold:*

$$\frac{\gamma_1}{\gamma_2} > \frac{\beta^2}{\alpha},$$

for when both agents are active (C1 holds); and

$$V > \frac{(1+\alpha)^3\beta^2\gamma_2}{\alpha^2}, \tag{14}$$

for when agents 2 only is active (C3 holds).

Proof. To prove the first inequality in the Proposition (when C1 holds and both agents are active), using the corresponding equilibrium marginal effort levels of agents in eq. (12) we get

$$\frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^{3/2}}{\sqrt{V}\alpha\beta^2\gamma_2} > \frac{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^{3/2}}{\sqrt{V}\alpha^2\gamma_1},$$

which after some algebra within the domain of the admissible values of the model parameters this simplifies to

$$\frac{\gamma_1}{\gamma_2} > \frac{\beta^2}{\alpha},$$

which proves the first part of the Proposition.

To prove the second inequality in the Proposition (when C3 holds and agent 2 only is active), using the corresponding equilibrium marginal effort levels of agents in eq. (12) we get

$$\frac{\alpha\left(\beta^2\gamma_2\right)^{1/3}}{\sqrt{\left((V\alpha^2)^{1/3}-\left(\beta^2\gamma_2\right)^{1/3}\right)^2}} < 1,$$

which again after some algebra within the domain of the admissible values of the model parameters this simplifies to

$$V > \frac{(1+\alpha)^3\beta^2\gamma_2}{\alpha^2},$$

which proves the second part of the Proposition. ■

This Proposition summarizes the effects of the implementation of the AA policy option on the levels of the marginal cost of competitive effort of the players. It shows, that if both agents are active or only agent 2 is active in the first stage of the game, then the *ex ante* weaker player - agent 2 may become the stronger one in the final stage of the game, meaning that in such a case the normative objective of the AA policy to create a level playing field, is totally missed: in equilibrium in the final stage of the game the *ex ante* weaker (discriminated) player becomes actually stronger than the *ex ante* stronger (non-discriminated) one. If both agents are active, then what is required for this to happen is the intensity of the affirmative action policy α high enough and/or the marginal cost of the learning effort of agent 2 γ_2 relative to the marginal cost of learning effort of agent 1, γ_1 low enough and/or the marginal cost of competitive effort *ex ante* of agent 2, β, low enough. A similar effect can be observed if only agent 2 is active in the first stage of the game. In this case agent 2 may become the stronger player in the final stage of the game when the marginal cost of the learning effort of agent 2 γ_2 is low enough and/or the marginal cost of competitive effort *ex ante* of agent 2, β is low enough. However, the role played by the intensity of the AA policy α is slightly more complex. α enters the condition (14) in a non-monotonic way: the RHS of the condition is decreasing for $\alpha < 2$, increasing for $\alpha > 2$, with a minimum when $\alpha = 2$. Therefore the levels of α that are either very low - close to 1, or very high - much above 2, make it less likely for agent 2 to become the stronger player in the final stage of the game. The intuition behind this last result is that with low levels of the intensity of the AA policy, the chances of agent 2 to win the contest are low anyway, therefore it not beneficial for them to incur the extra cost of learning, that would not improve their winning chances too much. In turn with high levels of the intensity of the AA policy, it simply doesn't make sense for agent 2 to invest in learning effort, as the AA policy will improve their relative performance anyway without them incurring any cost.

The results in Proposition 3 highlight the importance of the actual costs of acquiring skills (that is of education or learning) by individuals and their role played in designing effective AA programmes. They show that such programmes will not be efficient if they are designed in isolation based on the minority-group membership only and without taking into account the actual costs of acquiring skills.

3.5 Optimality of the Affirmative Action Policy

Contests may be used to meet different objectives. As contests create inherent incentives for the players to exert high level of competitive effort, they could be designed so that the level of the total competitive effort is maximized. In other settings the contest organizer may want to maximize the total level of learning effort of players. In this section we study the optimality of the AA policy of various intensity levels using as standards of comparison the sums of learning effort levels and competitive effort levels of players.

3.5.1 Maximization of Total Learning Effort

The equilibrium learning effort level of each contestant are dependent on the ex-ante announced intensity of the AA policy given by parameter α and the standard of comparison will therefore be expressed and denoted in the following way: $E_L^* = \sum_{i\in N} \epsilon_i^*(\alpha)$ for $\alpha \geq 1$. Using eq. (11), the equilibrium sum of learning effort levels given parameter α admits

$$E_L^* = \begin{cases} \frac{\alpha^2\beta^2\left(\alpha^2\gamma_1^2+\beta^2\gamma_2^2\right)}{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^3} V - 2, & \text{if} \quad C1, \\ \frac{\alpha^{2/3}\left(\left(V\beta^2\right)^{1/3}-\left(\alpha^2\gamma_1\right)^{1/3}\right)^2}{\beta^2\gamma_1^{2/3}} - 1, & \text{if} \quad C2, \\ \frac{\beta^{2/3}\left(\left(V\alpha^2\right)^{1/3}-\left(\beta^2\gamma_2\right)^{1/3}\right)^2}{\alpha^2\gamma_2^{2/3}} - 1, & \text{if} \quad C3, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

The following Proposition summarizes our results as regards the optimality of the AA policy when the objective of the contest holder is to maximize the total level of learning effort of players.

Proposition 4 *The total level of equilibrium learning effort E_L^* is maximized when the following conditions hold:*

a) for when both agents are active (C1 holds):

$$\alpha = \frac{\beta\sqrt{\gamma_2\left(\gamma_1-\gamma_2+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}}{\gamma_1}, \mathit{for}\, \beta \geq \sqrt{\frac{\gamma_1\left(\gamma_2-\gamma_1+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}{\gamma_2^2}},$$

$$\alpha = 1 \qquad \mathit{for}\ \beta < \sqrt{\frac{\gamma_1\left(\gamma_2-\gamma_1+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}{\gamma_2^2}}$$

b) when agent 1 only is active (C2 holds)

$$
\begin{array}{ll}
\textit{globally at } \alpha \to \infty, & \textit{if} \quad V \le \frac{\gamma_1}{\beta^2} \\
\begin{cases} \textit{locally at } \alpha = 1, \\ \textit{globally at } \alpha \to \infty, \end{cases} & \textit{if } \frac{\gamma_1}{\beta^2} < V < \frac{27\gamma_1}{\beta^2} \\
\begin{cases} \textit{locally at } \alpha = \beta\sqrt{\frac{V}{27\gamma_1}}, \\ \textit{globally at } \alpha \to \infty, \end{cases} & \textit{if} \quad V \ge \frac{27\gamma_1}{\beta^2},
\end{array}
$$

c) when agent 2 only is active (C3 holds)

$$
\begin{array}{ll}
\begin{cases} \textit{locally at } \alpha = \beta\sqrt{\frac{V}{27\gamma_1}}, \\ \textit{globally at } \alpha = 1 \end{cases} & \textit{if} \quad V \le \beta^2\gamma_2, \\
\textit{globally at } \alpha = \beta\sqrt{\frac{V}{27\gamma_1}}, & \textit{if } \beta^2\gamma_2 < V \le 27\beta^2\gamma_2 \\
\textit{globally at } \alpha = 1, & \textit{if} \quad V > 27\beta^2\gamma_2.
\end{array}
$$

Proof. To prove part a) of the Proposition (when C1 holds and both agents are active): using the corresponding equilibrium total learning effort level in (15) and differentiating with respect to α yields

$$\frac{2\alpha\beta^2 V\left(2\alpha^2\beta^2\gamma_1\gamma_2(\gamma_1-\gamma_2)+\beta^4\gamma_2^3-\alpha^4\gamma_1^3\right)}{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^4},$$

which is positive if

$$1 \le \alpha < \frac{\beta\sqrt{\gamma_2\left(\gamma_1-\gamma_2+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}}{\gamma_1}, \text{for } \beta > \sqrt{\frac{\gamma_1\left(\gamma_2-\gamma_1+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}{\gamma_2^2}},$$

negative if

$$
\begin{array}{ll}
\alpha > \frac{\beta\sqrt{\gamma_2\left(\gamma_1-\gamma_2+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}}{\gamma_1}, & \text{for } \beta > \sqrt{\frac{\gamma_1\left(\gamma_2-\gamma_1+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}{\gamma_2^2}}, \\
\alpha > 1, & \text{for } \beta = \sqrt{\frac{\gamma_1\left(\gamma_2-\gamma_1+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}{\gamma_2^2}}, \\
\alpha \ge 1, & \text{for } \beta < \sqrt{\frac{\gamma_1\left(\gamma_2-\gamma_1+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}{\gamma_2^2}},
\end{array}
$$

and equal to zero, if

$$\alpha = \frac{\beta\sqrt{\gamma_2\left(\gamma_1-\gamma_2+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}}{\gamma_1}, \text{for } \beta \ge \sqrt{\frac{\gamma_1\left(\gamma_2-\gamma_1+\sqrt{\gamma_1^2-\gamma_1\gamma_2+\gamma_2^2}\right)}{\gamma_2^2}}.$$

This can be rewritten as in the Proposition, proving its part a).

Part b) and c) of the Proposition (when either C2 or C3 holds and one agent only is active) follows directly the results for when only one of the players is active summarized in Conclusion 1 and 2, which define the intensity levels of the AA policy that induce the highest levels of the learning effort of players. As now the total learning effort level is equal to the effort level of the active agent, the same conditions define where the maximum of the total level of equilibrium learning effort is, which proves part b) and c) of the Proposition. ■

It follows from Proposition 4 that depending on the specific combination of the model parameters, there can either one (proper) global maximum, or one local and one global (proper) maximum(a), or no (proper) maximum at all. We have a clearly defined global maximum and no local maxima when both agents are active. However, this becomes more complicated when one agent only is active. As discussed earlier, when the discriminated agent - agent 2 is active only, there is both a (proper) global maximum, and a (proper) local maximum. The most complicated situation is when agent 1 is active only. In this case, in general there is no (proper) global maximum, and for some specific model parameters there may be some local maximum.

3.5.2 Maximization of Total Competitive Effort

In this section we focus on the optimality of the AA policy of various intensity levels using as a standard of comparison the sum of competitive effort levels of players. To allow analytical tractability, we will limit our considerations here only to the case when both agents are active (C1 holds). The equilibrium competitive effort level of each contestant is dependent on the intensity of the *AA* policy. Therefore the standard of comparison will be denoted in the following way: $E_C^* = \sum_{i\in N} e_i^*(\alpha)$ for $\alpha \geq 1$. Plugging the equilibrium marginal effort levels of agents for when C1 holds and both of them are active given in eq. (12) into eq. (7) produces

$$E_C^* = \frac{\alpha^3\beta^2\gamma_1\gamma_2\left(\alpha\gamma_1+\beta^2\gamma_2\right)}{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^{7/2}}V^{3/2}. \tag{16}$$

The following Proposition summarizes the conditions for the optimal level of the intensity of the *AA* policy with the objective to maximize the total level of competitive effort:

Proposition 5 *For when both agents are active (C1 holds), the total level of equilibrium competitive effort E_C^* is maximized when*

$$\begin{aligned}\alpha &= R(\alpha,3)\,, \mathit{for}\, \tfrac{\gamma_1}{\gamma_2} \leq \beta^2,\\ \alpha &= 1 \qquad \mathit{for}\, \tfrac{\gamma_1}{\gamma_2} > \beta^2,\end{aligned}$$

where $R(\alpha,3)$ is the third root of polynomial $R(\alpha)$:

$$R(\alpha) = -3\beta^4\gamma_2 - 4\beta^2\gamma_1\gamma_2\alpha + 4\beta^2\gamma_1\gamma_2\alpha^2 + 3\gamma_1^2\alpha^3.$$

Proof. Using the corresponding equilibrium total competitive effort level in eq. (16) and differentiating with respect to α yields

$$\frac{\alpha^2\beta^2\gamma_1\gamma_2\left(3\beta^4\gamma_2 - 4\alpha\left(\alpha-1\right)\beta^2\gamma_1\gamma_2 - 3\alpha^3\gamma_1^2\right)}{\left(\alpha^2\gamma_1+\beta^2\gamma_2\right)^{9/2}}V^{3/2},$$

which is positive if

$$\alpha < R(\alpha, 3), \text{for } \tfrac{\gamma_1}{\gamma_2} < \beta^2,$$

negative if

$$\begin{array}{ll} \alpha > R(\alpha, 3), & \text{for } \tfrac{\gamma_1}{\gamma_2} < \beta^2, \\ \alpha > 1 & \text{for } \tfrac{\gamma_1}{\gamma_2} = \beta^2, \\ \alpha \geq 1 & \text{for } \tfrac{\gamma_1}{\gamma_2} > \beta^2, \end{array}$$

and equal to zero, if

$$\alpha = R(\alpha, 3), \text{for } \tfrac{\gamma_1}{\gamma_2} \leq \beta^2,$$

where $R(\alpha, 3)$ is the third root of polynomial $R(\alpha)$

$$R(\alpha) = -3\beta^4\gamma_2 - 4\beta^2\gamma_1\gamma_2\alpha + 4\beta^2\gamma_1\gamma_2\alpha^2 + 3\gamma_1^2\alpha^3$$

This can be rewritten as in the Proposition, proving its part a). ■

It follows from Proposition 5 that when both agents are active, then there is a clearly defined global maximum of the total level of competitive effort. This maximum depends first of all on the relation between $\frac{\gamma_1}{\gamma_2}$ and β^2. If $\frac{\gamma_1}{\gamma_2} \leq \beta^2$, that is when the relative marginal cost of learning effort of agent 1 with respect to agent 2 is not higher than the square of the size of the discrimination level of agent 2 *ex ante* (as reflected by β)[14], then there is a maximum of the total level of competitive effort given by $\alpha = R(\alpha, 3) > 1$. This means that some degree of the intensity of the AA policy is required to induce the highest level of the total competitive effort. In the other case, that is when $\frac{\gamma_1}{\gamma_2} > \beta^2$,the maximum is at $\alpha = 1$, that is when the ET policy option is implemented. This means that implementation of the AA policy of any intensity ($\alpha > 1$) would actually be harmful to the level of the total competitive effort of the players, by reducing it.

4 Conclusions

In this paper we study a version of the Tullock model with two heterogenous players, in which one player is considered a discriminated one with a higher level of the marginal cost of competitive effort *ex ante*. The players are allowed to manipulate their original levels of marginal cost of competitive effort by investing in education. In this setup, our main objective is to investigate how different intensities of the affirmative action policy affect players' incentives modify their marginal cost of the competitive effort by investing in their learning effort.

We show that the value of the prize in the competition stage of the game plays a crucial role in incentivising the players to invest in their own learning effort. To guarantee that both of them will exert learning effort requires a high value of the prize. With a moderate prize value only one agent may invests in education, and with a low

[14]That could be the case when the level of discrimination of agent 2 ex ante β is high and/or when the marginal cost of learning effort of agent 2, γ_2 is relatively high as compared to the marginal cost of learning effort of agent 1, γ_1 (learning is costly for agent 2 in relative terms).

value – none of them. Our analysis demonstrates also that whenever a player invests in learning effort, the precise level of that investment, and the resulting level of the marginal cost of competitive effort varies as the intensity of the AA policy changes.

Over the wealth of the parameter space, the relation between the learning effort level and the intensity of the AA policy follows for both agents an inverted-U-shaped curve, with one distinct maximum in the intensity level, and those maxima are different for different players. This is precisely the case when both agents, 1 and 2, are active in the game. Additionally, what is interesting here, that in some cases those maxima may occur for when the intensity of the AA policy is zero, meaning that actually the ET policy is implemented. The exact combination of the model parameters when this happens is different for different players, but in general it is required that the level of discrimination of agent 2 *ex ante* is low and/or when the marginal cost of learning effort of agent 1 is relatively high as compared to the marginal cost of learning effort of agent 2.

When there is only one agent that is active, the relation between the learning effort level and the intensity of the AA policy becomes more complicated. When the active agent is agent 1, then there is no (proper) global maximum of the learning effort, and any proper maxima, if they exist, are local only. When agent 2 only is active, then there is always one (proper) global maximum, or both a (proper) global maximum and a (proper) local maximum. This global maximum may be when the intensity of the AA policy is zero, meaning again that actually the ET policy is implemented.

As a result of exerting learning effort, the levels of the parameters of marginal cost of competitive effort are reduced. We show that for some constellations of the model parameters, this reduction for the discriminated player - agent 2 is much higher than for the non-discriminated player - agent 1, which leads to a situation in which the *ex ante* weaker player actually becomes the stronger one in the final stage of the game. In such a case the objective of the AA policy is totally missed. If both agents are active, then this situation happens when the intensity of the affirmative action policy is high enough and/or the marginal cost of the learning effort of agent 2 relative to the marginal cost of learning effort of agent 1 is low enough, and/or the marginal cost of competitive effort *ex ante* of agent 2 is low enough. A similar situation can occurs if only agent 2 is active in the first stage of the game, and both - the marginal cost of the learning effort of agent 2 and/or the marginal cost of competitive effort *ex ante* of agent 2 - are low enough. However, in this cases the role played by the intensity of the AA policy is slightly more complex than previously and the intensity levels that are either very low, or very high, make it less likely for agent 2 to become the stronger player in the final stage of the game.

Using our model we also investigated the problem of the optimality of the contests from the point of view of the contest holder. As the standard of comparison of various policies we used the total level of the learning effort and the total level of the competitive effort. In the former case, we showed that there is a clearly defined global maximum and no local maxima when both agents are active. For small values of the level of the marginal cost of competitive effort *ex ante* of agent 2 this maximum occurs when the

ET policy is implemented. However, when the discriminated agent - agent 2 is active only, there is both a (proper) global maximum, and a (proper) local maximum of the total learning effort levels. When agent 1 is active only, in general there is no (proper) global maximum. In the latter case (the standard of comparison is the total level of the competitive effort), for when both agents are active there is a clearly defined global maximum. The analysis shows that when the level of discrimination of agent 2 *ex ante* is high and/or when the marginal cost of learning effort of agent 2 is relatively high as compared to the marginal cost of learning effort of agent 1, then to induce the highest level of the total competitive effort some degree of the intensity of the AA policy is required. Otherwise, the maximum occurs for when the ET policy is implemented.

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